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# HOLON WORKSHOP IN COMPLEX ANALYSIS

DEPARTMENT OF APPLIED MATHEMATICS  
HOLON ACADEMIC INSTITUTE OF TECHNOLOGY

15 APRIL 2005

## PROGRAMME

- 09:00 – 09:15      **REGISTRATION AND REFRESHMENTS**
- 09:15 – 09:20      **WELCOME**
- 09:20 – 10:00      **PROF. HERSHEL M. FARKAS** (HEBREW UNIVERSITY, JERUSALEM)  
THE THETA FUNCTION IN COMPLEX ANALYSIS
- 10:00 – 10:45      **PROF. SAMUEL KRUSHKAL** (BAR-ILAN UNIVERSITY, RAMAT-GAN)  
TWO CONJECTURES ON GRUNSKY INEQUALITIES
- ◆ COFFEE BREAK ◆
- 11:00 – 11:40      **PROF. VLADIMIR GOL'DSHTEIN** (BEN-GURION UNIVERSITY, BEER-SHEVA)  
EMBEDDING THEOREMS AND BOUNDARY VALUE PROBLEMS FOR  
ROUGH DOMAINS
- 11:45 – 12:25      **PROF. DAUD BSHOUTY** (TECHNION, HAIFA)  
ON CROFOOT-SARASON CONJECTURE FOR HARMONIC POLYNOMIALS
- ◆ COFFEE BREAK ◆
- 12:40 – 13:20      **PROF. MIKHAIL SODIN** (TEL-AVIV UNIVERSITY, RAMAT-AVIV)  
AREA OF THE POSITIVITY SET OF HARMONIC POLYNOMIALS
- 13:20 – 14:00      **PROF. DAVID SHOIKHET** (ORT BRAUDE COLLEGE, KARMIEL)  
A FLOWER STRUCTURE OF BACKWARD FLOW INVARIANT DOMAINS  
FOR SEMIGROUPS OF HOLOMORPHIC FUNCTIONS

**ORGANIZING COMMITTEE: E. YAKUBOV (HAIT), A. GOLBERG (BAR-ILAN UNIVERSITY),  
U. SREBRO (TECHNION)**

## ABSTRACTS

### ◆ **HERSHEL M. FARKAS, THE THETA FUNCTION IN COMPLEX ANALYSIS**

IN THIS TALK I SHALL SHOW BY EXAMPLE HOW THE THETA FUNCTION PLAYS A PROMINENT ROLE IN COMPLEX ANALYSIS. THE EXAMPLES I CHOOSE WILL RUN FROM CONFORMAL MAPPING OF RECTANGLES ONTO DISCS, PICARD'S THEOREM ON ENTIRE FUNCTIONS, AND MODULI OF COMPACT RIEMANN SURFACES.

### ◆ **SAMUEL KRUSHKAL, TWO CONJECTURES ON GRUNSKY INEQUALITIES**

THE TALK ADDRESSES QUANTITATIVE ESTIMATING QUASICONFORMAL EXTENSIONS OF HOLOMORPHIC FUNCTIONS ON THE BASE OF FUNDAMENTAL GRUNSKY INEQUALITIES. AN IMPORTANT CONJECTURE HERE WAS STATED BY JÜRGEN MOSER IN 1985 AND CONCERNS THE RELATION BETWEEN THE GRUNSKY CONSTANT AND TEICHMÜLLER NORM OF EXTENSIONS. THERE IS ALSO ANOTHER CONJECTURE STATING SOMEWHAT IN CONTRAST TO ABOVE. BOTH CONJECTURES WILL BE DISCUSSED. THE RESEARCH IS JOINT WITH REINER KÜHNAU.

### ◆ **VLADIMIR GOL'DSHTEIN, EMBEDDING THEOREMS AND BOUNDARY VALUE PROBLEMS FOR ROUGH DOMAINS**

EXISTENCE AND COMPACTNESS OF EMBEDDING OPERATORS OF SOBOLEV SPACE TO LEBESGUE SPACE AND CORRESPONDING FUNCTIONAL SPACES ON THE BOUNDARY OF DOMAINS WILL BE DISCUSSED. THESE DOMAINS INCLUDES, IN PARTICULAR, INCLUDES DOMAINS WHOSE BOUNDARIES ARE LIPSCHITZ MANIFOLDS, BUT MUCH LARGER CLASSES OF DOMAINS INCLUDES IN ABOVE CLASSES. APPLICATIONS TO ELLIPTIC BOUNDARY PROBLEMS ARE MENTIONED. SOME GENERALIZATIONS OF QUASICONFORMAL HOMEOMORPHISMS (P-QUASICONFORMAL HOMEOMORPHISMS) WAS USED FOR THIS STUDY.

### ◆ **DAOUD BSHOUTY, ON CROFOOT-SARASON CONJECTURE FOR HARMONIC POLYNOMIALS**

LET  $F(z) = P_N(z) + \bar{z}$  BE A HARMONIC POLYNOMIAL, WHERE  $P_N(z)$  IS A COMPLEX POLYNOMIAL OF DEGREE  $N$ . IT IS KNOWN THAT SUCH A POLYNOMIAL HAS AT MOST  $3N - 2$  ZEROES IN THE PLANE. IT IS NOT KNOWN IF THIS RESULT IS SHARP. THE CROFOOT-SARASON CONJECTURE SUGGESTS AN EXAMPLE OF SHARPNESS. NAMELY A POLYNOMIAL  $Q_N(z)$  IS AN EXAMPLE IF THERE EXIST POINTS  $z_1, z_2, \dots, z_{N-1}$  SUCH THAT SIMULTANEOUSLY  $P_N(z_j) + \bar{z}_j = 0$  AND  $P'_N(z_j) = 0$  FOR  $J = 1, 2, \dots, N - 1$ . WE EXAMINE THIS CONJECTURE FOR SMALL VALUES OF  $N$ .

### ◆ **MIKHAIL SODIN, AREA OF THE POSITIVITY SET OF HARMONIC POLYNOMIALS**

SUPPOSE  $F(z)$  IS A POLYNOMIAL OF DEGREE  $D > 1$  VANISHING AT THE ORIGIN. THEN THE AREA OF THE SET OF POINTS IN THE UNIT DISC WHERE  $\text{Im} F(z)$  IS POSITIVE CANNOT BE SMALLER THAN  $\text{const}/\log D$ , AND THIS ESTIMATE IS SHARP. IF TIME PERMITS WE DISCUSS VARIOUS EXTENSIONS OF THIS RESULT, APPLICATION TO LOCAL ASYMMETRY OF LAPLACE-BELTRAMI EIGENFUNCTIONS, AND MOCK-EQUIDISTRIBUTION OF THE ARGUMENT OF ENTIRE FUNCTIONS. THIS IS A JOINT WORK WITH F. NAZAROV AND L. POLTEROVICH.

◆ **DAVID SHOIKHET, A FLOWER STRUCTURE OF BACKWARD FLOW INVARIANT DOMAINS FOR SEMIGROUPS OF HOLOMORPHIC FUNCTIONS**

IN THIS WORK, WE STUDY CONDITIONS WHICH ENSURE THE EXISTENCE OF BACKWARD FLOW INVARIANT DOMAINS FOR SEMIGROUPS OF HOLOMORPHIC SELF-MAPPINGS OF A SIMPLY CONNECTED DOMAIN  $D$ . MORE PRECISELY, THE PROBLEM IS THE FOLLOWING. GIVEN A ONE-PARAMETER SEMIGROUP (SEMIFLOW)  $S$  ON  $D$ , FIND A SIMPLY CONNECTED SUBSET  $\Omega$  IN  $D$  SUCH THAT EACH ELEMENT OF  $S$  IS AN AUTOMORPHISM OF  $\Omega$ , IN OTHER WORDS, SUCH THAT  $S$  FORMS A ONE-PARAMETER GROUP (FLOW) ON  $\Omega$ . ON THE WAY TO SOLVING THIS PROBLEM, WE PROVE AN ANGLE DISTORTION THEOREM FOR STARLIKE AND SPIRALLIKE FUNCTIONS WITH RESPECT TO INTERIOR AND BOUNDARY POINTS. THIS IS A JOINT WORK WITH MARK ELIN, DEPARTMENT OF MATHEMATICS, ORT BRAUDE COLLEGE AND LAWRENCE ZALCMAN, DEPARTMENT OF MATHEMATICS, BAR-ILAN UNIVERSITY.