



# **Holon Workshop** **in Applied Harmonic Analysis** **and Approximation Theory**

**30.04.2004**

- 9-30 - 10-00 **Registration and refreshments**
- 10-00 - 10-40 **Prof. Yuri Brudnyi, Technion**  
**Dr. Inna Kozlov, EORD, Technion and HAIT**  
Algorithms of Non-linear Approximation
- 10-40 - 11-20 **Prof. Amir Averbuch, Tel-Aviv University**  
3-D Discrete X-Ray Transform
- Coffee break**
- 11-30 - 12-10 **Prof. Nira Dyn, Tel-Aviv University**  
TBA
- 12-10 - 12-50 **Prof. Maria Skopina, St. Petersburg University**  
On constructing multivariate regular wavelets
- Refreshments**
- 13-20 - 14-00 **Prof. Valery Zheludev, Tel-Aviv University**  
Interpolatory frames in signal space
- 14-00 - 14-40 **Dr. Yossef Ben-Ezra, HAIT**  
Chaos and self-similarity in nonlinear physical systems

### *Organizing committee*

**A. Averbuch, Tel-Aviv University, A. Golberg, Bar-Ilan University,  
I. Kozlov, Technion and HAIT and E. Yakubov, HAIT**

## Algorithms of Non-Linear Approximation

Professor Yuri Brudnyi (Technion) and Dr. Inna Kozlov (EORD, Technion and HAIT)

In this lecture we present an overview of algorithms and results for  $N$ -term nonlinear approximation by polynomials, refinable functions and wavelets.

## 3-D Discrete X-Ray Transform

Professor Amir Averbuch (Tel-Aviv University)

The analysis of 3-D discrete volumetric data becomes increasingly important as computation power increases. 3-D analysis and visualisation applications are expected to be especially relevant in areas like medical imaging and non-destructive testing, where elaborated continuous theory exists. However, this theory is not directly applicable to discrete datasets. Therefore, we have to establish theoretical foundations for discrete analysis tools that will replace the existing inexact discretisations, which have been based on the continuous theory. For the discrete theory we want to preserve the concepts, properties and main results of the continuous theory.

In this talk we present a discretisation of the continuous X-ray transform for discrete 3-D images. Our definition of the discrete X-ray transform is shown to be exact and geometrically faithful as it uses summation along geometric lines without arbitrary interpolation schemes. We derive a discrete Fourier slice theorem, which relates our discrete X-ray transform with the Fourier transform of the underlying image, and then use this Fourier slice theorem to derive a constructive algorithm that computes the discrete X-ray transform using  $O(n^4 \log n)$  operations. Finally, we show that our discrete X-ray transform is invertible. (Joint work with Yoel Shkolnisky).

## Image Compression by Linear Splines over Adaptive Triangulations

Professor Nira Dyn (Tel-Aviv University)

A new method for image compression is proposed. The method is based on the approximation of an image, regarded as a function, by a linear spline over an adapted triangulation,  $D(Y)$ , which is the Delaunay triangulation of a small set  $Y$  of significant pixels. The linear spline minimizes the distance to the image, measured by the mean square error, among all linear splines over  $D(Y)$ . The significant pixels in  $Y$  are selected by an adaptive thinning algorithm, which recursively removes less significant pixels in a greedy way, using a sophisticated criterion for measuring the significance of a pixel. The proposed compression method combines the approximation scheme with a customized scattered data coding scheme. We demonstrate that our compression method outperforms JPEG2000 on two geometric images and performs competitively with JPEG2000 on three popular test cases of real images.

## On Constructing Multivariate Regular Wavelets

Professor Maria Skopina (St. Petersburg University)

We consider biorthogonal multivariate compactly supported wavelets with matrix dilation generated by a couple of scaling function. For regularity of a wavelet function, the partial derivatives of its Fourier transform should vanish at the origin. We present a necessary and sufficient condition on scaling functions under which all wavelet functions possess this property. The condition is given in explicit form convenient for implementations.

## Interpolatory Frames in Signal Space

Professor Valery Zheludev (Tel-Aviv University)

Recently frames or redundant expansions of signals attracted a considerable interest of researchers working in signal processing. As the requirement of one-to-one correspondence between the signal and its transform coefficients is dropped, there is more freedom in the design and implementation of the frame transforms. The frame expansions of signals demonstrate resilience to the quantization noise and to the coefficients losses. Thus, frames may serve as a tool for error correction by the transmission of signals through lossy channels. Additional adaptation abilities of the overcomplete representation of signals has a potential to succeed in feature extraction and identification of signals. An important class of frames is the frames generated by filter banks. Actually, under some relaxed conditions, a perfect reconstruction oversampled filter bank produces the frame expansion. We present a new family of frames, which are generated by perfect reconstruction filter banks consisting of linear phase filters. The filter banks are designed on the base of the polynomial and discrete interpolatory and quasi-interpolatory splines. Each filter bank comprises one interpolatory symmetric low-pass filter and two high-pass filters, one of which is also interpolatory and symmetric. The second high-pass filter may be symmetric or antisymmetric. These filter banks generate the analysis and synthesis scaling functions and pairs of framelets. We introduced the concept of semi-tight frame. While in the tight frame all the analysis waveforms coincide with their synthesis counterparts, in the semi-tight frame we can vary the second framelets making them different for the synthesis and the analysis cases. By this means we can switch the vanishing moments from the synthesis to the analysis framelets or to add smoothness to the synthesis framelet. We constructed dual pairs of frames, where all the waveforms are symmetric and all the framelets have the same number of vanishing moments. Although most of the designed filters have infinite impulse response, they allow fast implementation via recursive procedures. The waveforms are well localized in the time domain despite their infinite support. The frequency response of the designed filters is flat.

## Chaos and Self-Similarity in Nonlinear Physical Systems

Dr. Yossef Ben-Ezra (HAIT)

We present the chaotic behaviour and self-organization mechanisms of different physical systems. We show the fractal nature of ATM/IP communication networks. The Wavelet Transform is used for study of the self-similarity of these systems.